Trigonometria évfolyamdolgozat

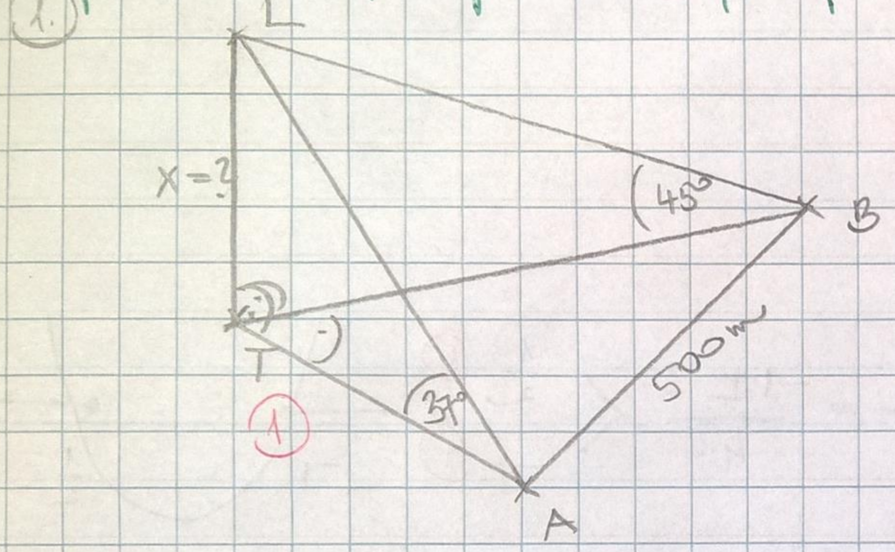
Javítási és pontozási útmutató

2017. December 6.

TRIGONOMETRIA   11. évf. FAK.   Évfolyamdolgozat
2017. 12. 06.

56 ponttól 5; 45 ponttól 4; 34 ponttól 3; 23 ponttól 2.

1.



$\operatorname{tg} 37° = \frac{x}{AT}$ 1 $\longrightarrow AT = \frac{x}{\operatorname{tg} 37°}$ 1 $= 1{,}327x$ 1

$\operatorname{tg} 45° = \frac{x}{BT}$ $\qquad BT = \frac{x}{\operatorname{tg} 45°} = \frac{x}{1} = x$ 2

$AT^2 + BT^2 = 500^2$

$\left(\frac{x}{\operatorname{tg} 37°}\right)^2 + x^2 = 500^2$ 1 $\quad / \cdot \operatorname{tg}^2 37°$

$x^2 + \operatorname{tg}^2 37° \cdot x^2 = 250.000 \cdot \operatorname{tg}^2 37°$

$x^2 = \frac{250\,000 \cdot \operatorname{tg}^2 37°}{1 + \operatorname{tg}^2 37°} = 90545{,}33$

$x = 300{,}91$ m 2

V: A léggömb 300,91 m magasan lebeg. 1

10 pont

2.

$3 \cdot \sin x + 4 \cdot \cos x \qquad (x \in \mathbb{R})$

$/ : \sqrt{3^2+4^2} = 5$ ①

$5 \cdot \left(\frac{3}{5} \sin x + \frac{4}{5} \cos x \right)$ ② $= 5 \cdot \sin (x + 53{,}13°)$ ②

$\frac{3}{5} = \cos \alpha$

$\alpha = 53{,}13°$

	ÉK:
$\sin x$	$[-1; 1]$
$\sin (x + 53{,}13°)$	$[-1; 1]$ ①
$5 \cdot \sin (x + 53{,}13°)$	$[-5; 5]$ ②

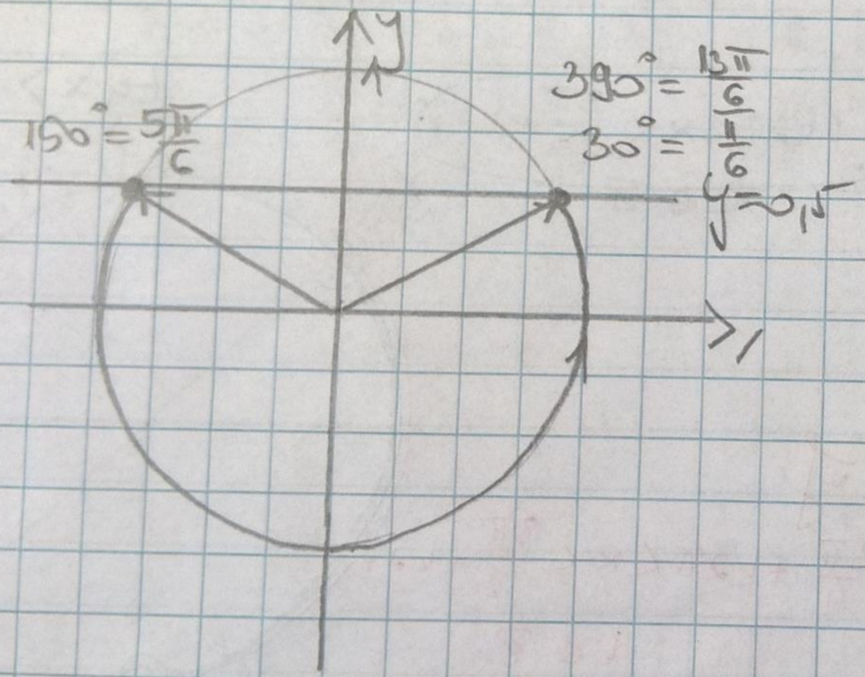
8 pont

3.

a) $\sin x \cdot \cos x \leq 0{,}25 \quad / \cdot 2$

$2 \sin x \cos x \leq 0{,}5$ 1

$\sin 2x \leq 0{,}5$ 2



$$\frac{5\pi}{6} + k \cdot 2\pi \leq 2x \leq \frac{13\pi}{6} + k \cdot 2\pi \;;\; k \in \mathbb{Z}$$ 2 1

$/ : 2$

$$\frac{5\pi}{12} + k \cdot \pi \leq x \leq \frac{13\pi}{12} + k \cdot \pi$$ 1

7 pont

MATH just do it.

b) $\cos 2x \le \sin x$

$\cos^2 x - \sin^2 x \le \sin x$ (1)

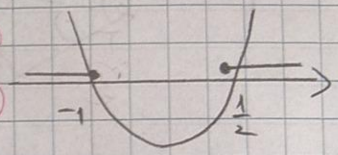
$1 - 2\sin^2 x \le \sin x$

$0 \le 2\sin^2 x + \sin x - 1$ (2)

$\sin x = a$

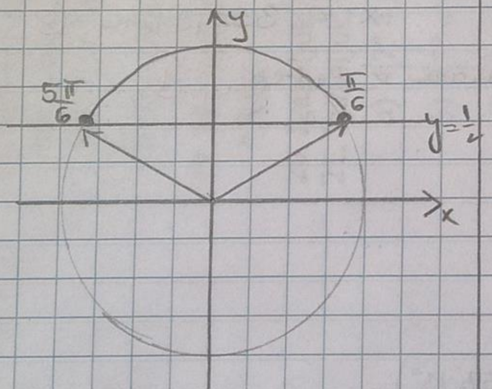
$0 \le 2a^2 + a - 1$

$a_{1,2} = \frac{-1 \pm \sqrt{1 - 4\cdot 2\cdot(-1)}}{4} = \frac{-1 \pm 3}{4} \Big\langle \begin{matrix} \frac{1}{2} \; (1) \\ -1 \; (1) \end{matrix}$



$a \ge \frac{1}{2}$ vagy $a \le -1$

$\sin x \ge \frac{1}{2}$ (1)



$\frac{\pi}{6} + k_1 2\pi \le x \le \frac{5\pi}{6} + k_1 2\pi$ (2)

$k_1 \in \mathbb{Z}$

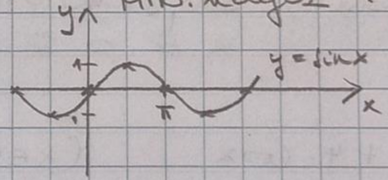
$\sin x \le -1$ (1)

$\Updownarrow$

$\sin x = -1$ (1)

$\sin x \in [-1; 1]$ (1)

Min. helyek:



$x = -\frac{\pi}{2} + k_2 \cdot 2\pi$ (1)

$k_2 \in \mathbb{Z}$

12 pont

MATH just do it.

c) $\frac{\cos x}{1+\operatorname{tg} x} < 0$ ÉT: $x \neq \frac{\pi}{2} + k\cdot\pi;\ k \in Z$ $1+\operatorname{tg} x \neq 0$

1

$\operatorname{tg} x \neq -1$

$x \neq -\frac{\pi}{4} + k\cdot\pi;\ k \in Z$

TÖRT < 0, HA SZÁMLÁLÓJA ÉS NEVEZŐJE KÜL. ELŐJELŰ:

I. $\cos x > 0$ és $1+\operatorname{tg} x < 0$

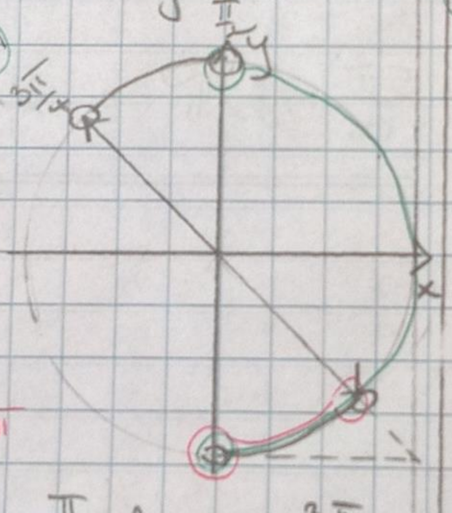
⇓

4

$-\frac{\pi}{2} + k2\pi < x < \frac{\pi}{2} + k2\pi$

$k \in Z$

$\operatorname{tg} x < -1$



ÖJ:

$\frac{3\pi}{2} + k2\pi < x < \frac{7\pi}{4} + k2\pi$

$k \in Z$

$\frac{\pi}{2} + k2\pi < x < \frac{3\pi}{4} + k2\pi$

$k \in Z$

II. $\cos x < 0$ és $1+\operatorname{tg} x > 0$

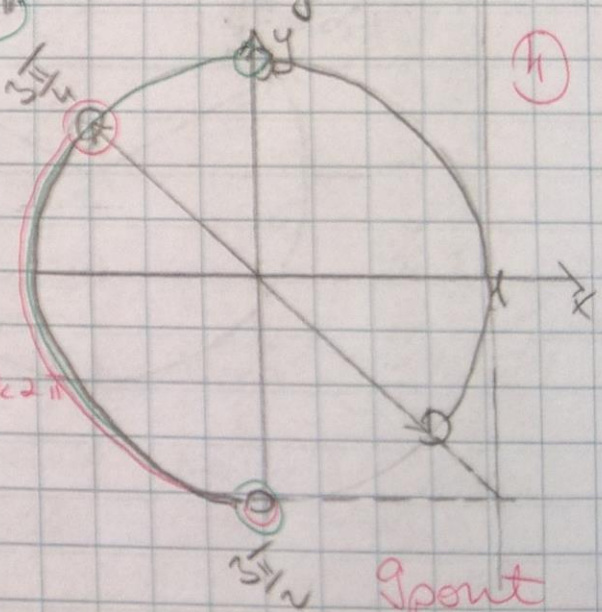
⇓

$\frac{\pi}{2} + k2\pi < x < \frac{3\pi}{2} + k2\pi$

$k \in Z$

$\operatorname{tg} x > -1$

1



ÖJ:

$\frac{3\pi}{4} + k2\pi < x < \frac{3\pi}{2} + k2\pi$

$k \in Z$

9pont

just do it.

4.

$\sin x + \cos x = c$

a) $\sin x \cdot \cos x = ?$

$(\sin x + \cos x)^2 = c^2$ 1

$\sin^2 x + 2\sin x \cdot \cos x + \cos^2 x = c^2$

$1 + 2 \cdot \sin x \cdot \cos x = c^2$

1

$\sin x \cdot \cos x = \frac{c^2 - 1}{2}$ 2

4 point

b) $\sin x + \cos x = c \quad / \sqrt{1^2 + 1^2} = \sqrt{2}$

$\frac{1}{\sqrt{2}} \sin x + \frac{1}{\sqrt{2}} \cos x = \frac{c}{\sqrt{2}}$

$\frac{1}{\sqrt{2}} = \cos \alpha$

$\alpha = 45° = \frac{\pi}{4}$

$\sin x \cdot \cos \frac{\pi}{4} + \cos x \cdot \sin \frac{\pi}{4} = \frac{c}{\sqrt{2}}$

$\sin\left(x + \frac{\pi}{4}\right) = \frac{c}{\sqrt{2}} \qquad -1 \le \sin\left(x + \frac{\pi}{4}\right) \le 1$

$\Downarrow$

$-1 \le \frac{c}{\sqrt{2}} \le 1$

$-\sqrt{2} \le c \le \sqrt{2}$

MATH just do it.

MATH
just do it.

VAGY

$$2 \cdot \sin x \cdot \cos x = c^2 - 1$$

$$\sin 2x = c^2 - 1 \quad 2 \qquad -1 \le \sin 2x \le 1 \quad 1$$

$$-1 \le c^2 - 1 \le 1 \quad 1$$

$$0 \le c^2 \le 2 \quad 1$$

$$-\sqrt{2} \le c \le \sqrt{2} \quad 1$$

6 pont

MATH
just do it.

5.

a) $\sin 3x = \cos 5x$

$\sin 3x = \sin\left(\frac{\pi}{2} - 5x\right)$ 2 vagy $\cos 5x = \cos\left(\frac{\pi}{2} - 3x\right)$

$3x = \frac{\pi}{2} - 5x + k_1 \cdot 2\pi$; $k_1 \in \mathbb{Z}$ 1 $\quad /+5x$

$8x = \frac{\pi}{2} + k_1 \cdot 2\pi \quad /:8$

$\underline{x_1 = \frac{\pi}{16} + k_1 \cdot \frac{\pi}{4}}$ 2

$3x + \frac{\pi}{2} - 5x = \pi + k_2 \cdot 2\pi$; $k_2 \in \mathbb{Z}$ 1

$-2x + \frac{\pi}{2} = \pi + k_2 \cdot 2\pi \quad /-\frac{\pi}{2}$

$-2x = \frac{\pi}{2} + k_2 \cdot 2\pi \quad /:(-2)$

$\underline{x_2 = -\frac{\pi}{4} - k_2 \cdot \pi}$ 2

8 pont

b) $\cos x - \sin^2 x = -0{,}4$

$\cos x - (1 - \cos^2 x) = -0{,}4$ 1 $/ \times (-1) / +0{,}4$

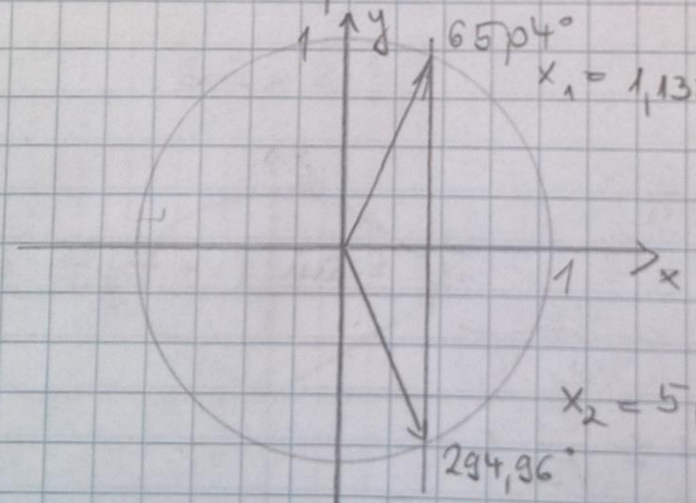
$\cos^2 x + \cos x - 0{,}6 = 0$ 2

$\cos x = a$

$a^2 + a - 0{,}6 = 0$

$$a_{1,2} = \frac{-1 \pm \sqrt{1 - 4 \cdot 1 \cdot (-0{,}6)}}{2} = \frac{-1 \pm 1{,}8439}{2} \begin{cases} 0{,}42195 & 1 \\ -1{,}42195 & 1 \end{cases}$$

$\cos x = 0{,}42195$



$x_1 = 1{,}1352 + k_1 \cdot 2\pi$ 1

$k_1 \in \mathbb{Z}$

$x_2 = 5{,}1480 + k_2 \cdot 2\pi$ 1

$k_2 \in \mathbb{Z}$

$\cos x = -1{,}42195$

$-1 \le \cos x \le 1$

nem ad, megoldást 1

8 pont

MATH just do it.

c)

$$2 \cdot \sin^2 x + \sin^2 2x = 2$$

$$2 \cdot \sin^2 x + 4 \cdot \sin^2 x \cdot \cos^2 x = 2 \cdot (\sin^2 x + \cos^2 x) \quad (1) \quad (1)$$

$$2 \sin^2 x + 4 \sin^2 x \cdot \cos^2 x = 2 \sin^2 x + 2 \cos^2 x$$

$$4 \cdot \sin^2 x \cdot \cos^2 x - 2 \cos^2 x = 0$$

$$2 \cdot \cos^2 x (2 \sin^2 x - 1) = 0 \quad (1)$$

$$\cos x = 0 \quad (1) \qquad \sin^2 x = \frac{1}{2} \quad (1)$$

$$\Downarrow$$

$$x_1 = \frac{\pi}{2} + k_1 \cdot \pi \; ; \; k_1 \in \mathbb{Z} \quad (1) \qquad \sin x = \frac{\sqrt{2}}{2} \quad (1) \qquad \sin x = -\frac{\sqrt{2}}{2} \quad (1)$$

$$\Downarrow$$

$$\left.\begin{array}{l} x_2 = \frac{\pi}{4} + k_2 \cdot 2\pi \; ; \; k_2 \in \mathbb{Z} \\ x_3 = \frac{3\pi}{4} + k_3 \cdot 2\pi \; ; \; k_3 \in \mathbb{Z} \end{array}\right\} (1) \qquad \left.\begin{array}{l} x_4 = \frac{5\pi}{4} + k_4 \cdot 2\pi \; ; \; k_4 \in \mathbb{Z} \\ x_5 = \frac{9\pi}{4} + k_5 \cdot 2\pi \; ; \; k_5 \in \mathbb{Z} \end{array}\right\} (1)$$

10 pont

MATH just do it.

MATH

just do it.

(6.)

$a : b = 2 : 3$

$\gamma = 30^\circ$

$T = 150\,cm^2$

$c = ?$

$R = ?$

10 pont

$T = \frac{a b \sin\gamma}{2}$ (1)

$150 = \frac{2x \cdot 3x \cdot \sin 30^\circ}{2}$

$150 = \frac{3}{2} x^2$ (1)

$x^2 = 100$

$|x| = 10 \Rightarrow x = 10$ (2)

$a = 20\,cm$; $b = 30\,cm$ (1)

$c^2 = 20^2 + 30^2 - 2 \cdot 20 \cdot 30 \cos 30^\circ$ (1)

$c = 16{,}15\,cm$ (2)

$R = \frac{c}{2 \sin\gamma}$ (1) $= \frac{16{,}15}{2 \cdot \sin 30^\circ} = 16{,}15$ (1)

Mi volt ebben olyan nehéz???